MATH 10B
Section 211
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Student:
SID:
Tue 4/9/19

Quiz 10

True/False - No explanation needed. (For each: 1 point if correct, 0 points if not answered, -1 points if incorrect)

1. For any continuous or discrete random variable X with a well-defined mean μ and variance σ^2, it is true that $\mu^2 + \sigma^2 = E(X^2)$ True/False

   True. This is a rephrasing of the statement $E(X^2) = \mu^2 + \sigma^2$

2. Chebyshev's inequality guarantees that at least 75% of a distribution is within two standard deviations of the mean. True/False

   True. The inequality gives us $P(\mu - 2\sigma < X < \mu + 2\sigma) \geq 1 - \frac{1}{4} = 75\%$

Problems - Needs justification.

1. Compute the
   (a) mean
   (b) variance
   (c) mode

   of the random variable with PDF $f(x) = \frac{3}{2x^{2.5}}$ for $x \geq 1$ and $f(x) = 0$ otherwise. (10 points)

For the mean,

$$\int_1^\infty x \cdot \frac{3}{2x^{2.5}} dx = \int_1^\infty \frac{3}{2x^{1.5}} dx = \frac{3}{2} \cdot \left(-\frac{2}{x^{.5}}\right)\Big|_0^\infty = 3$$

For variance

$$\int_1^\infty x^2 \cdot \frac{3}{2x^{2.5}} dx = \int_1^\infty \frac{3}{2x^{.5}} dx = \frac{3}{2} \cdot (2x^{.5})\big|_0^\infty = \infty$$

$\infty - 9 = \infty$, so variance of this RV is infinite.

Mode is the value that has the highest PDF. In our case as x increases, $f(x)$ decreases, so $x = 1$ is the mode, as it is the left side of the nonzero part of our distribution.