

True/False - No explanation needed. (For each: 1 point if correct, 0 points if not answered, -1 points if incorrect)

1. When using a random variable X and one experiment with it to estimate a parameter θ , we compare all values of $L(x|\theta)$ for $X = x$ fixed and θ varying.

True/False

True.

2. The formula $(\ln f(\theta))' = \frac{f'(\theta)}{f(\theta)}$ makes it possible to shift the process from finding critical points for the origin $f(\theta)$ to finding critical points of $\ln(f(\theta))$. True/False

True. In order to make this shift, we need to use the fact that the maxima of a function f are the same as the maxima of $\ln f$ because natural log is an increasing function; we can then simply use the chain rule (as shown in this question) to determine the zeros of the derivative and thus the critical points.

Problems - Needs justification.

1. The population of an Emperor Penguin colony is known to be normally distributed, with unknown mean and standard deviation 2,000. If you measure the population of 25 colonies, what is the probability that the sample mean is within 400 of the true mean? Express your answer in terms of z-scores.

The sample mean is a normal random variable with mean μ and standard deviation $2,000/\sqrt{25} = 400$. Therefore this probability is

$$\int_{\mu-2000}^{\mu+2000} \frac{1}{400\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2(400)^2}}$$

by substituting $u = \frac{x-\mu}{400}$ we get

$$\frac{1}{\sqrt{2\pi}} \int_{-400/400}^{400/400} e^{-u^2/2} du = 2z(1)$$