

**True/False** - No explanation needed. (For each: 1 point if correct, 0 points if not answered, -1 points if incorrect)

1. For an exponential distribution with PDF  $ce^{-cx}$  for  $x \geq 0$ , the maximum likelihood parameter  $\hat{c}$  is unbiased. True/False  
False. This is equivalent to saying that  $E\frac{1}{X} = c$  for an exponential distribution, which is false.
2. Testing a two sided alternative leads to calculating probabilities of the form  $P(|X| \geq |r|)$  or  $P[|Z| \geq |z|]$ . True/False  
False. This assumes that our distribution is symmetric around 0, which is not necessarily true.

**Problems** - Needs justification.

1. Judie measures the size of butterflies. The wingspan of each type of butterfly is normally distributed with a standard deviation of 2 mms. Judie's null hypothesis is that a certain butterfly is a skipper, which has average wingspan of 26 mms. Her alternative hypothesis is that it is a butterfly with a **bigger** wingspan with significance level  $\alpha = 0.04$ . She measures a wingspan of 30 mms.
  - (a) What is the rejection region associated with this problem?
  - (b) What is Judie's conclusion?(10 points)

The rejection region is the area such that  $P(X > r_0) = \alpha$ , so in this case it would be the region such that  $P(X > r_0) = \alpha$ , so  $P(\frac{X-\mu}{\sigma} > \frac{r_0-\mu}{\sigma}) = \alpha$ , so  $.5-z(\frac{r_0-\mu}{\sigma}) = .04$ . By looking at the z-table, we see that  $\frac{r_0-\mu}{\sigma} \approx 1.75$ .

Therefore  $r_0 \approx 29.5$  and the rejection region is  $[29.5, \infty)$ .

30 is in the rejection region, so Judie will refute the null hypothesis. We could also calculate the p-value and show that it is less than  $\alpha$ .