

## Quiz 13 Solution

**True/False** - No explanation needed. (For each: 1 point if correct, 0 points if not answered, -1 points if incorrect)

1. The recurrence relationship  $\Gamma(n) = (n - 1)\Gamma(n - 1)$  and the fact that  $\int_0^\infty e^{-t} dt = 1$  implies that  $\Gamma(n) = (n - 1)!$  for all  $n \geq 1$ . True/False

True. We can think about this inductively. The second property gives us  $\Gamma(1) = \int_0^\infty e^{-t} dt = 1$  and the first implies that if this equality is true for  $n$  it is true for  $n + 1$ .

2. In a t-distribution, the degrees of freedom  $\nu$  are 1 fewer than the number of measurements  $x_1, \dots, x_n$  because, even though  $\sigma_0$  may be unknown, it is fixed for  $X$ , thereby yielding a quadratic relation between the  $x_i$ 's and the sample mean  $x$ . True/False

False. This is the correct number of degrees of freedom, but not for the correct reason. For example,  $\sigma_0$  is not fixed for  $x$ . Rather it depends on the random samples of  $X$ .

**Problems** - Needs justification.

1. I have a coin that I believe to be fair. If I flip it 20 times and get 15 heads, can I reject my null hypothesis with significance level  $\alpha = .05$ ? Use a  $\chi^2$  statistic. (10 points)

To compute our distance from the expected value, we have

$$\frac{(15 - 10)^2}{10} + \frac{(5 - 10)^2}{10} = 5$$

The  $\chi^2$  statistic is 3.84, so we reject the null hypothesis that this is a fair coin.