MATH 10B | Student:
Section 211 | SID:
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Quiz 2 Solution

True/False - No explanation needed. (For each: 1 point if correct, 0 points if not answered, -1 points if incorrect)

1. The symmetry of permutations can be seen in the identity $P(n,k) = P(n, n-k)$ for all integer n, k. True/False

   False: For example $P(3,2) \neq P(3,1)$. However we have $C(n,k) = C(n, n-k)$.

2. The basic combinatorial relation satisfied by binomial coefficients that makes it possible to identify all numbers in Pascal's triangle as some binomial coefficients can be written as: $\binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n}{k}$. True/False

   True: It reflects the fact that a number in Pascal's triangle is the sum of the two numbers above it.

Problems - Need justification.

1. How many numbers must be selected from $\{1,2,3,4,5,6,7,8\}$ to guarantee that at least one pair of these numbers adds up to 9? (5 points)

   The pairs of numbers that add up to 9 are $\{\{1,8\},\{2,7\},\{3,6\},\{4,5\}\}$. If we have any two numbers from one of these sets, then we are done. If we consider these sets as "holes", there are 4 "holes" and we need to guarantee there is one hole with two numbers in it, so we need 5 numbers.

2. What is the coefficient of x^7y^9 in $(3x-2y)^{16}$? You don't need to write the actual integer. A formula will suffice. (5 points)

   We get a contribution of 3^7 from the x part and $(-2)^9$ from the y part. From the binomial theorem, we add a coefficient of $\binom{16}{7}$, so our final answer is $-(3^7 \cdot 2^9 \cdot \binom{16}{7})$