MATH 10B
Section 205
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Quiz 4 Solution

Student:
SID:
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True/False - No explanation needed. (For each: 1 point if correct, 0 points if not answered, -1 points if incorrect)

1. In the Odd-Pie Fight problem, it must be the case that two people throw pies at each other. True/False

   True. The two closest people throw pies at each other.

2. An infinite geometric series $a + ar + ar^2 + ar^3 + \ldots$ converges to $\frac{a}{1-r}$ if and only if $r \neq 1$. True/False

   False, we require $|r| < 1$

Problems - Need justification.

1. Use mathematical induction to prove that if n is a positive integer,

$$\sum_{\{a_1,\ldots,a_k\}\subseteq\{1,2,\ldots n\}} \frac{1}{a_1 \cdot a_2 \cdots a_k} = n$$

   (Here the sum is over all nonempty sets of the set of the n smallest positive integers). (10 points)

   Base Case: 1=1

   Inductive step: Assume that the equality is true for n, we will prove it for $n+1$. We split our sum into two parts. Sum 1, which we call S_1, sums all terms that do not include $n+1$

$$S_1 = \left(\sum_{\{a_1,\ldots,a_k\}\subseteq\{1,2,\ldots n\}} \frac{1}{a_1 \cdot a_2 \cdots a_k} \right) = n$$

   by the inductive hypothesis. Sum 2, S_2 includes all terms that contain $n+1$. Therefore

$$S_2 = \left(\frac{1}{n+1} S_1 \right) + \frac{1}{n+1} = \frac{n+1}{n+1} = 1$$

   so

$$S_1 + S_2 = n + 1$$