Quiz 5 Solution

True/False - No explanation needed. (For each: 1 point if correct, 0 points if not answered, -1 points if incorrect)

1. The statement

$$B_1 \cap B_2 \cap \ldots \cap B_n = \emptyset \text{ necessarily implies } B_i \cap B_j = \emptyset \text{ for every } 1 \leq i, j \leq n$$

   is false, but the implication in the opposite direction is true. True/False

   True. We could have, for example $B_1 = \overline{B_2}$. In this case $B_1 \cap B_3$ is not necessarily $\emptyset$, but $B_1 \cap B_2 \cap \ldots \cap B_n = \emptyset$. In the opposite direction, in fact if **any** pair has empty intersection then the entire intersection is also empty.

2. $P(A|B)$ is at most $P(B)$, considering that in $P(A|B)$ we are assuming the occurrence of B. True/False

   False, considering we divide by the probability $P(B)$ in $P(A|B)$. This would be true if it were $P(A \cap B)$ instead of $P(A|B)$.

Problems - Needs justification.

1. A space probe near Neptune communicates with earth using bit strings. Suppose that in its transmissions it sends a 1 one-third of the time and a 0 two-thirds of the time. When a 0 is sent, the probability that it is received incorrectly (as a 1) is 0.1. When a 1 is sent, the probability that it is received correctly is 0.8, and the probability that it is received incorrectly (as a 0) is 0.2. What is the probability that a 0 was transmitted, given that a 0 was received. Give your answer as a fraction in reduced terms. (10 points)

   We use Bayes' Theorem. Call R the probability that a 0 was received, and S the probability a 0 was sent. We get

$$P(S|R) = \frac{P(R|S)P(S)}{P(R|S)P(S) + P(R|\overline{S})P(\overline{S})} = \frac{.9 \cdot \frac{2}{3}}{.9\frac{2}{3} + .2 \cdot \frac{1}{3}} = \frac{6}{6 + \frac{2}{3}} = \frac{9}{10}$$