MATH 10B
Section 211
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Quiz 5 Solution

Student:
SID:
Tue 2/26/19

True/False - No explanation needed. (For each: 1 point if correct, 0 points if not answered, -1 points if incorrect)

1. Assuming that $P(A), P(B) > 0$, then

$$P(A|B) = P(B|A) \text{ implies } P(A) = P(B)$$

True/False

False. We get $\frac{P(A\cap B)}{P(B)} = \frac{P(A\cap B)}{P(A)}$, but this implies that $P(A) = P(B)$ only if $P(A \cap B) > 0$, namely A and B are not disjoint.

2. Assume E is an event from a sample space S, and $F_1, \ldots, F_n$ are mutually exclusive events such that $\bigcup_{i=1}^n F_i = S$. Moreover $P(E) > 0$ and $P(F_i) > 0$ for $i = 1, 2, \ldots, n$. Then

$$P(F_j|E) = \frac{P(E|F_j)P(F_j)}{\sum_{i=1}^n P(E|F_i)P(F_i)}$$

.
True/False

True. This is the generalized Bayes theorem, where we are breaking down

$$P(E \cap \overline{F_j}) = \sum_{i\neq j} P(E \cap F_i)$$

Problems - Needs justification.

1. 4/5 of mail sent to an email address is personal and 1/5 is spam. When a personal email is sent, the probability it is blocked by the address' spam filter is 5%. When spam is sent, the probability it is blocked is 90%. What is the probability that a message that is blocked by the spam filter is personal? Write as a fraction in lowest terms. (10 points)

Call A the probability that sent mail is personal, and B the probability mail is blocked. By Bayes' Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\overline{A})P(\overline{A})} = \frac{.05 \cdot \frac{4}{5}}{.05 \cdot \frac{4}{5} + .9 \cdot \frac{1}{5}} = \frac{\frac{1}{25}}{\frac{1}{25} + \frac{9}{50}} = \frac{2}{11}$$