MATH 10B
Section 205
GSI: Theo McKenzie

Quiz 6 Solution

Student:
SID:
Tue 3/5/19

True/False - No explanation needed. (For each: 1 point if correct, 0 points if not answered, -1 points if incorrect)

1. If X is the random variable of the product of two dice rolls, then $|X^{-1}(4)| = 3$. True/False

   True. The elements of the preimage are $\{(1,4),(2,2),(4,1)\}$

2. If A and B are disjoint, then they are independent. True/False

   False. In fact, as long as the probability of each is nonzero then they will be dependent. $P(A \cap B) = 0$, but we could have $P(A)P(B) > 0$

Problems - Needs justification.

1. How many people n are necessary to have at least a $1/2$ chance that at least three people are born on a Tuesday? Assume a birthday occurs on a given day of the week with probability $1/7$. We don't need an actual number, just an equation with n as a variable. (10 points)

   Assume there are n people. We check the complement. The probability that there are no people born on Tuesday is $\frac{6^n}{7^n}$. The probability that there is exactly one person born on Tuesday is $\binom{n}{1}(\frac{1}{7})\frac{6^{n-1}}{7^{n-1}}$, as there are $\binom{n}{1}$ ways to place the person born on Tuesday. Similarly for 2 people we have $\binom{n}{2}(\frac{1}{7^2})(\frac{6^{n-2}}{7^{n-2}})$. Therefore our formula is that we would need n at least the value satisfying

$$1 - \frac{6^n}{7^n} - \binom{n}{1}\frac{6^{n-1}}{7^n} - \binom{n}{2}\frac{6^{n-2}}{7^n} = \frac{1}{2}$$