

Quiz 7 Solution

True/False - No explanation needed. (For each: 1 point if correct, 0 points if not answered, -1 points if incorrect)

1. X is a Poisson random variable and Y is a geometric variable counting the number of failures. X and Y have the same range. True/False

True. In both cases the range is $\{0, 1, 2, \dots\}$

2. The St. Petersburg Paradox proves that the expectation of a random variable does not have to be finite. True/False

True. This is an example of a problem with infinite expectation.

Problems - Needs justification.

1. Out of the 50,000 people in Hoboken, we expect there to be 10 werewolves.
 - (a) Use **two different** probability distributions to estimate the PMF function of X , where X is the number of werewolves in Hoboken. You only need write a formula, not calculate the actual value.
 - (b) Do you expect the PMF functions to be close to each other or not? Why?

(10 points)

- a) First way is binomial. The expectation of the binomial distribution is np , so p must be $\frac{10}{50,000} = \frac{1}{5,000}$. Therefore the PMF is

$$P(X = k) = \binom{50,000}{k} \left(\frac{1}{5,000}\right)^k \left(\frac{4,999}{5,000}\right)^{n-k}$$

Second way is Poisson. Here we're given $\lambda = 10$, so

$$P(X = k) = \frac{10^k e^{-10}}{k!}$$

- b) We expect these functions to be close, as the Poisson approximates the binomial if p is small and n is large.