

True/False - No explanation needed. (For each: 1 point if correct, 0 points if not answered, -1 points if incorrect)

1. The PDF $f(x) = \frac{1}{3x^{4/3}}$ for $1 \leq x \leq \infty$ and $f(x) = 0$ otherwise, has finite median but infinite mean. True/False

True. This is similar to the Pareto distribution. $\int_1^\infty xf(x)dx = \infty$ but $\int_1^8 \frac{1}{3x^{4/3}} = 1/2$

2. Shifting the bell-shaped PDF $f(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ to the right by 2 units results in another PDF $g(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{(x-2)^2}{2}}$ centered at 2. True/False

True. By changing the exponent to $x - 2$ we are shifting our function to the right.

Problems - Needs justification.

1. Assume that the PDF of x is

$$f(x) = \sqrt{\frac{2}{\pi}}e^{-x^2/2}$$

for $0 \leq x \leq \infty$ and 0 otherwise. What is the mean of this random variable? (10 points)

$$\int_{-\infty}^{\infty} xf(x)dx = \sqrt{\frac{2}{\pi}} \int_0^{\infty} xe^{-x^2/2} = \sqrt{\frac{2}{\pi}} \left(-e^{-\frac{x^2}{2}} \Big|_0^{\infty} \right) = \sqrt{\frac{2}{\pi}}$$