

## Quiz 1 Solutions

**True/False** - No explanation needed. (For each: 1 point if correct, 0 points if not answered, -1 points if incorrect)

1. To find how many natural numbers between 1 and  $n$  are divisible by  $d$  we calculate the fraction  $\frac{n}{d}$  and round up in order not to miss any numbers. True/False

False. We have to round down, not round up.

2.  $A \times B \times C$ . for some sets  $A, B$ , and  $C$  is another set made of all possible triplets  $(x, y, z)$  where  $x, y$ , and  $z$  are any elements of any of the three sets. True/False

False.  $x$  must be an element of  $A$ ,  $y$  must be an element of  $B$  and  $z$  must be an element of  $C$ .

**Problems** - Need justification.

1. How many positive integers less than or equal to 100 are odd or perfect squares? (5 points)

We use the principal of inclusion exclusion. There are 50 odd numbers less than or equal to 100 and 10 perfect squares. There are 5 perfect squares that are odd less than or equal to 100. Therefore our answer is  $50 + 10 - 5 = 55$ .

2. A child is building a stack of blocks marked by digits and a blank block. How many different towers of four blocks can they build if they use 3 digit blocks and the blank block. For the digit blocks, assume there are 10; each is marked by exactly one of the 10 digits. (5 points)

I accepted two answers to this problem. I meant without replacement, but worded it poorly. If we assume there is one of each type of digit block, then there are four options for where to place the blank block, then  $10 \cdot 9 \cdot 8$  options for the other places. Therefore the answer is  $4 \cdot 10 \cdot 9 \cdot 8 = 2880$ .

If we assume the digit blocks occur with replacement, then there are  $10 \cdot 10 \cdot 10$  options for the other places, meaning our answer is  $4 \cdot 10 \cdot 10 \cdot 10 = 4000$ .