MATH 10B
Section 211
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Student:
SID:
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Quiz 1 Solutions

True/False - No explanation needed. (For each: 1 point if correct, 0 points if not answered, -1 points if incorrect)

1. When X and Y are mutually disjoint, we need only add the sizes of X and Y to find the size of $X \cup Y$. True/False

   True. As these are mutually disjoint sets, we do not need to worry about double counting the intersection.

2. To find how many possibilities there are for *exactly* one of X and Y occur, we take $|X \cup Y|$ and then subtract the size of the intersection. True/False

   True. The elements we are looking for are those in $X \cup Y$ that are not in $X \cap Y$.

Problems - Need justification.

1. How many functions f are there from $\{1, 2, 3, 4, 5\}$ to $\{1, 2, 3\}$ if $f(2) \leq 2$
   (5pts)

   For each element of $x \in \{1, 2, 3, 4, 5\}$, we have three options for $f(x)$, except for $f(2)$, for which there are only two options. Therefore we have five slots, with 3 options for each except for 1 option in one slot. Therefore there are a total of $3^4 \cdot 2 = 162$ options.

2. How many positive integers between 50 and 250 are divisible by either 3 or 7?

   Here it doesn't matter if we consider or exclude 250 and 50, but let's assume that they're included. The number of positive integers less than or equal to 250 that are divisible by 3 is $\lfloor \frac{250}{3} \rfloor$. The number of positive integers less than or equal to 49 that are divisible by 3 is $\lfloor \frac{49}{3} \rfloor$. Therefore the number of integers between 250 and 50 that are divisible by 3 is $\lfloor \frac{250}{3} \rfloor - \lfloor \frac{49}{3} \rfloor$. By the principle of inclusion exclusion, we add the number of integers divisible by 3 and the number of integers divisible by 7, then subtract the number of integers divisible by the least common multiple. This gives us

   $$\left\lfloor \frac{250}{3} \right\rfloor - \left\lfloor \frac{49}{3} \right\rfloor + \left\lfloor \frac{250}{7} \right\rfloor - \left\lfloor \frac{49}{7} \right\rfloor - \left\lfloor \frac{250}{21} \right\rfloor + \left\lfloor \frac{49}{21} \right\rfloor = 83 - 16 + 35 - 7 - 11 + 2 = 86$$