Discussion on Probability and Independence

1. Find each of the following probabilities when n independent Bernoulli trials are carried out with probability of success p.

   a) The probability of no failures

   b) The probability of at least one failure

   c) The probability of at most one failure

   d) The probability of at least two failures

2. Two dice are rolled.

   a) Are the events that the first die rolled is a 1 and that the sum of the two dice is a 7 independent?

   b) Are the events that the first die rolled is a 1 and that the sum of the two dice is a 6 independent?

3. Find the smallest number of people you need to choose at random so that the probability that at least two of them were both born on February 1st exceeds $1/2$. Assume the probability of being born on any given day is $1/366$.

4. Let E be the event that a randomly generated bit string of length three contains an odd number of 1s, and let F be the event that the string starts with 1. Are E and F independent?

5. If E and F are independent, are E and $\overline{F}$ necessarily independent? Prove or disprove.

6. Find the probability that a randomly generated bit string of length 10 does not contain a 0 if bits are independent and if

   a) a 0 bit and a 1 bit are equally likely.

   b) the probability that a bit is a 1 is 0.6

   c) the probability that the ith bit is a 1 is $1/2^i$ for $i = 1, 2, 3, \ldots, 10$.