I. Random variables

1. Suppose that we roll two die and let X be equal to the maximum of the two rolls. Find $P(X \in 1, 3, 5)$ and draw the PMF for X.
   Solution: First we draw the PMF. We calculate $P(X = x)$ by counting the number of ways we can roll two die so that the maximum is x and then dividing by the total number of possibilities, which is 36. So for instance, the only way to get $X = 1$ is if we roll (1, 1) and hence $P(X = 1) = 1/36$. Then $P(X = 2) = f(1, 2); (2, 2); (2, 1)/36 = \frac{3}{36}$. Thus, we have that:
   $f(1) = \frac{1}{36}, f(2) = \frac{3}{36}, f(3) = \frac{5}{36}, f(4) = \frac{7}{36}, f(5) = \frac{9}{36}, f(6) = \frac{11}{36}$
   We draw the PMF with stalks at 1 through 6 of those respective heights. Then $P(X \in 1, 3, 5) = P(X = 1) + P(X = 3) + P(X = 5) = 5/12$

2. When rolling two die, let Y be equal to the first die roll. Are X, Y independent random variables?
   Solution: No. Intuitively, if we know that the first die roll is a 6, then the maximum has to be a 6. Mathematically writing that, we see that$P(X = 6 \cap Y = 6) = 1/6$, but $P(Y = 6)P(X = 6) = 11/216$ so $P(X = 6, Y = 6) \neq P(Y = 6)P(X = 6)$

3. I flip a fair coin n times. Let X be the number of heads I get. Draw the PMF for X. What if the probability of heads is p?
   Solution: This is a binomial distribution. The range is $0, 1, 2, \ldots, n$. Then $P(X = k)$ is the number of ways to get x heads over the total number of ways so $P(X = k) = \frac{\binom{n}{k}}{2^n}$. For a generalized binomial this is $\binom{n}{k}p^k(1-p)^{n-k}$

4. I roll two fair four sided die with sides numbered $1 - 4$. Let X be the product of the two numbers rolled. Find the range of X and draw the PMF for X.
   Solution: The range is all products of two numbers in 1, 2, 3, 4. This is 1, 2, 3, 4, 6, 8, 9, 12, 16. We calculate:

x	1	2	3	4	6	8	9	12	16
$f(x)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

5. Diana is at the gym shooting half court shots on the basketball court. The probability that she makes each shot is .1 Assuming she shoots until she misses, what is the probability she takes more than 10 shots?
   Solution: This is a geometric distribution. The probability that we do more than ten trials is the complement of finishing in at most 10 trials. Therefore our answer is
   $$1 - .9 - (.1)(.9) - (.1)^2(.9) - \ldots - (.1)^9(0.9)$$
   By using the formula for a finite geometric sequence, we see this is
   $$1 - \frac{.1 - (.1)^{10}.9}{1 - .1} = (.1)^{10}$$
   Alternatively, we can think about this as the probability she misses each of the first 10 shots, which is $(.1)^{10}$

6. A coin is biased so that the probability of heads is 2/3. What is the probability that exactly four heads come up when the coin is flipped seven times, assuming that the flips are independent. **Solution**: $X\ Bin(7, 2/3)$, $P(X = 4) = \binom{7}{4}(\frac{2}{3})^4(\frac{1}{3})^3 = 560/2187$