1 Variance

1. **(original)** Which of the following situations are possible? Explain.
   (a) A random variable X_1 has expected value 1 and variance $\frac{1}{2}$.
   (b) A random variable X_2 has expected value 1 and variance $-\frac{1}{2}$.
   (c) A random variable X_3 has expected value 1 and variance 1000000.
   (d) A random variable X_4 has expected value 1, variance 4, and standard error 4.
   (e) A Poisson random variable X_5 has expected value 1 and variance 2.
   (f) Two inepdendent random variables X and Y have $E[X] = 1$, $E[Y] = 1$, $\text{Var}[X] = 2$, and $\text{Var}[Y] = 3$, and their sum $X + Y$ has $E[X + Y] = 2$ and $\text{Var}[X + Y] = 4$.
2. **(original)** I flip a fair coin twice. Let X denote the number of heads and Y denote the number of tails.
   (a) What is $\text{Var}[X]$?
   (b) What is $\text{Var}[Y]$?
   (c) What is $\text{Var}[X + Y]$?
3. **(inspired by pset 19 #1)** A binomial random variable X has mean 20 and variance 4. Can you find n and p for this random variable? What is $P(X = 2)$?
4. **(original)** Suppose playing a gambling game at one casino has an expected value of -1 dollars and a variance of 50 cents. At a pricier casino across the street, the exact same game (with the same probabilities of various levels of payouts) is played except the amount of money you pay to play is 10 times as high and all payouts are 10 times as high as well. What is the expected value and variance of this game at the pricier casino?

2 Covariance and limits of random variables

6. **(based on pset 20 #1)** Let X_1, X_2, and X_3 denote the numbers that come up on three rolls of a fair *four-sided* die. Let $X = X_1 + X_2 + X_3$, $Y = X_1 + X_2$, $\overline{X} = X/3$, $\mu = E[X_1]$, and $\sigma^2 = \text{Var}[X_1]$.
   (a) What are the ranges of X_1, X_2, X_3, X, $\overline{X}$, and Y?
   (b) Find the variances of X_1, X_2, X_3, X, $\overline{X}$, and Y.
   (c) Find $\text{Cov}[X_1, Y]$ and $\text{Cov}[X_3, Y]$.
   (d) Are X_1 and Y independent? Why or why not? What about X_3 and Y?
   (e) Let $\overline{\mu} = E[\overline{X}]$ and $\overline{\sigma}^2 = \text{Var}[\overline{X}]$. If $Z = \frac{\overline{X} - \overline{\mu}}{\overline{\sigma}}$, find $E[Z]$ and $\text{Var}[Z]$.
   (f) What is the pmf of Z? Sketch a picture.

All problems labeled "inspired by pset" adapted from the problem sets. Problem set 19 is adapted from previous 10B HWs written by Anna Seigal. Problem set 20 is new for this year's iteration math 10B.