I. PDFs and CDFs

1. (a) Call $f(x) = \sin(x) + \frac{1}{2\pi}$ for $0 \leq x \leq 2\pi$ and $f(x) = 0$ otherwise. Verify that $f(x)$ is a valid PDF.
   (b) What is the CDF of this function?
   (c) What is $P(X \leq \pi/2)$?

2. (a) Call $f(x) = ae^{-ax}$ for $0 \leq x \leq \infty$ and $f(x) = 0$ otherwise, where $a > 0$ is a constant. Verify that $f(x)$ is a valid PDF.
   (b) What is the CDF of this function?
   (c) What is $P(1 \leq X \leq 2)$?

3. (a) Call $f(x) = \frac{1}{\pi}\frac{1}{\sqrt{1-z^2}}$ for $-1 \leq x \leq 1$. Verify that $f(x)$ is a valid PDF.
   (b) What is the CDF?
   (c) What is $P(-\frac{1}{\sqrt{2}} \leq X \leq 0)$?

4. (a) Call $f(x) = \frac{1}{x}$ for $e \leq x \leq e^2$. Verify that $f(x)$ is a valid PDF.
   (b) What is the CDF?
   (c) What is $P(e^{1.5} \leq X \leq e^{1.75})$?

5. What PDF has CDF $\sin(x)$ for $0 \leq x \leq \pi/2$?

6. I flip a fair coin 15 times, represented by the random variables $X_1, \ldots, X_{15}$. The average of these variables is $\overline{X}$. What is the probability that X and $\overline{X}$ are within one standard error of their expectations? (Calculate exactly!). What is a way to estimate this (do the estimation).

7. (Review Question)!

   In the Ancient Greek Attic Calendar, there are 10 months in a year. How many people chosen randomly do we need to ensure that the probability that at least two people are born in the same Attic month is at least 1/2? Assume that birthdays are uniform.