Chebyshev

1. Take the Pareto distribution $\frac{4}{x^5}$ for $x \geq 1$. What is the probability $P(\mu - 3\sigma < X < \mu + 3\sigma)$? What is the bound Chebyshev gives us?

$$\mu = \int_1^\infty \frac{4}{x^4} dx = -\left.\frac{4}{3x^3}\right|_1^\infty = \frac{4}{3}$$
$$\int_1^\infty \frac{4}{x^3} dx = -\left.\frac{4}{2x^2}\right|_1^\infty = 2$$

so $\sigma^2 = 2\frac{16}{9} = \frac{2}{9}$. Therefore $\sigma = \frac{\sqrt{2}}{3}$.

The integral we then take is

$$\int_1^{\frac{4}{3}+\sqrt{2}} \frac{4}{x^5} dx = -\left.\frac{1}{x^4}\right|_1^{\frac{4}{3}+\sqrt{2}} \approx 98.2\%.$$

Chebyshev gives us that this is at least $1 - \frac{1}{3^2} \approx 88.9\%$

2. The RV X is a Laplace distribution with PDF $\frac{1}{2}e^{-|x|}$. What is $P(|X| > 3)$? What bound does Chebyshev give us?

This is $\int_3^\infty e^{-x} dx = e^{-3} \approx 5\%$. $\mu = 0, \sigma = \sqrt{2}$, so Chebyshev gives us $P(|X| > 3) \leq \frac{\sigma^2}{9} \approx 22\%$

3. Bubbles the clown blows up 100 balloons an hour, with a variance of 16 balloons. What is a lower bound on the probability Bubbles blows between 94 and 106 balloons? $\mu = 100$, $\sigma = 4$.

Therefore this is $P(\mu - 1.5\sigma < X < \mu + 1.5\sigma) \geq 1 - \frac{1}{2.25} \approx 56\%$, by Chebyshev's inequality.

4. What distribution that we have studied best models the random variable X, where X is the number of emails Nicole receives in an hour, assuming that she receives an average of 4? What is a formula for the exact value $P(X > 10)$? How can we estimate the probability $P(X > 10)$? This is poisson. The exact formula will be

$$1 - \sum_{k=0}^{10} \frac{e^{-4}4^k}{k!}$$

An approximation can be made as follows.

$$P(X > 10) = P(X \geq 11) = P(X - \mu \geq 7) = P(|X - \mu| \geq 7) \leq \frac{1}{3.5^2} = .82\%$$

5. For the random variable X with PDF $f(x) = ce^{-cx}$ for $x \geq 0$, what is $P(\mu - 2\sigma < X < \mu + 2\sigma)$? What bound does Chebyshev give us?

Here we have $\mu = \sigma = \frac{1}{c}$. Therefore this is

$$\int_0^{\frac{3}{c}} ce^{-cx} dx = e^{-cx}\Big|_0^{\frac{3}{c}} = 1 - e^{-3} \approx 95\%$$

Chebyshev gives us a lower bound of $1 - \frac{1}{2^2}$, which is 75%.

6. Packer High School's high jump team jumps an average of 180 cm with a standard deviation of 8 cm. Assuming the distribution is normally distributed, what is the probability that someone on the team jumps to a height of at least 2 meters?

By converting to $\frac{X-\mu}{\sigma}$ we get $P(X \geq 200) = P(\frac{X-\mu}{\sigma} \geq 2.5) = .5 - z(2.5) = .5 - .4938 = .6\%$