Worksheet 24

1 Z-Test

1. An infomercial claims that a miracle drug will cause you to grow all your hair back. There are 25 brave participants and surprisingly 7 people regrew their hair. If normally 10% of people regrow their hair, can you say that this drug worked?

   Solution: We would expect that 10% of people will regrow their hair with standard deviation $= \sigma = \sqrt{p(1-p)} = \sqrt{0.1 \times 0.9} = 0.3$. The central limit theorem says that with a sample of 25 people, we expect that 10% of people regrow their hair with a standard deviation of $\sigma/\sqrt{n} = \frac{0.3}{\sqrt{25}} = 0.06$. There are $7/25 = 28\%$ who regrew their hair. The z score is $z(|0.28 - 0.1|/0.06) = Z(3) < \alpha$. Therefore, we can reject the null hypothesis and say that this drug does help you grow your hair back.

2. You flip a coin 100 times and get 55 heads. Can you say that it is biased towards heads? (use $\alpha = 0.05$)

   Solution: The null hypothesis is that the coin is unbiased and hence $p = 0.5$. The standard deviation $= \sigma = \sqrt{p(1-p)} = 0.5$. Thus, the central limit theorem tells us that the percentage of coin flips we get is approximately normally distributed with a standard deviation of $\sigma/\sqrt{n} = \frac{0.5}{\sqrt{100}} = 0.05$. There are $55/100 = 55\%$ of heads. The z score is $z(|0.55 - 0.5|/0.05) = Z(1) > \alpha$. Therefore, we cannot reject the null hypothesis and say that this drug does help you grow your hair back.

3. An infomercial claims that a miracle drug will cause you to grow all your hair back. There are 100 brave participants and this time 20 people regrew their hair. If normally 10% of people regrow their hair, can you say that this drug worked?

   Solution: We would expect that 10% of people will regrow their hair with standard deviation $= \sigma = \sqrt{p(1-p)} = \sqrt{0.1 \times 0.9} = 0.3$. The central limit theorem says that with a sample of 100 people, we expect that 10% of people regrow their hair with a standard deviation of $\sigma/\sqrt{n} = \frac{0.3}{\sqrt{100}} = 0.03$. There are $20/100 = 20\%$ who regrew their hair. The z score is $z(|0.2 - 0.1|/0.03) = Z(3.33) < \alpha$. Therefore, we can reject the null hypothesis and say that this drug does help you grow your hair back.

2 T-Test

Concept:What is the t-statistic, and what is it used for?

For a sample of size n, the t-statistic is a measure of how far the sample mean $\overline{X}$ lies from the hypothesized population mean μ_0, measured in units of the standard error in the mean $s/\sqrt{n}$. The t-statistic is given by

$$T_{n-1} = \frac{\overline{X} - \mu_0}{s/\sqrt{n}}$$

It is used during hypothesis testing to determine whether the sample data are compatible with the null hypothesis. It usually deal with the case that has small sample size.

Worksheet 24

1. The heart rates of 40 patients in an ICU have mean 95.3beats/min and standard deviation 16.9 beats/min. Are heart rates from ICU patients unusual given normal heart rate has mean of 72 beats/min with a significance of .01?

   (a) What is the degree of freedom?

   Solution: degree of freedom $= n - 1 = 40 - 1 = 39$

   (b) What is the t-statistic?

   Solution: $T_{n-1} = \frac{95.3-72}{16.9/\sqrt{40}} \approx 8.72$

   So this will be much smaller than $\alpha = .01$

2. When individuals become infected with malaria the parasite consumes red blood cells, causing the patient to become anemic. Red blood cell concentration was measured in a group of 18 mice, 10 days after infection, giving the following data, in (cells $\times 10^6/\mu L$)

3.2	4.5	3.7	3.3	2.5	3.6
3.3	3.6	4.2	3.5	2.0	3.1
2.3	2.9	6.9	4.5	7.3	3.0

   Assume the data is from a normal distribution. Does the data provide evidence that the mean red blood cell concentration differs from (3.0×10^6) cells/μL at the $\alpha = .01$ significance level?

   We have the degrees of freedom as 17, and the average is 3.75. Therefore the statistic is $\frac{(3.75-3)\sqrt{18}}{1.4} \approx 2.27$. So the probability is $2(.5 - t(2.27)) \approx .03$ which is bigger than α.