

.5cm

1. How many functions are there from  $\{1, 2, 3, 4, 5\}$  to  $\{1, 2, 3\}$  such that  $f(3)$  is odd?
2. How many solutions are there to  $x_1 + x_2 + x_3 \leq 20$  such that  $3 \leq x_1 \leq 6, 1 \leq x_2, 3 \leq x_3$ ?
3. Prove that the number of diagonals of a convex  $n$ -gon is  $n(n-3)/2$ . Test this out for  $n = 3$  and  $n = 4$  to make sure you believe this.
4. How many integers from 50 to 100 (inclusive) are divisible by 7 but not 4?
5. How can you line up 10 men and 5 women such that no two women stand next to each other?
6. Show that if there are 101 people of different heights standing in a line, it is possible to find 11 people in the order they are standing in the line with heights that are either increasing or decreasing.
7. When a test for steroids is given to soccer players, 98% of the players taking steroids test positive and 12% of the players not taking steroids test positive. Suppose that 5% of soccer players take steroids. What is the probability that a soccer player who tests positive takes steroids?
8. Let  $X_n$  be the random variable that equals the number of tails minus the number of heads when  $n$  fair coins are flipped. What is the expected value of  $X_n$ ? What is the variance of  $X_n$ ?