

More combinations and permutations

1. How many permutations of the letters $ABCDEFGG$ contain:
 - (a) the string ACE ?
 - (b) the strings AG and FCB ?
 - (c) the strings AB , DC , and GE ?
 - (d) the strings ACB and GFE ?
2. Ten women and eight men are on the faculty of a mathematics department at a school.
 - (a) How many ways are there to select a committee of five members of the department if at least one woman must be on the committee?
 - (b) How many ways are there to select a committee of five members of the department if at least one man and at least one woman must be on the committee?

Binomial coefficients

3. What is the coefficient of x^6y^{10} in the expansion of $(2x + 5y)^{16}$?
4. The row of Pascal's triangle containing the binomial coefficients $\binom{10}{k}$, $0 \leq k \leq 10$, is:

1 10 45 120 210 252 210 120 45 10 1

Use Pascal's identity to produce the row immediately preceding this row in Pascal's triangle.

5. Show that if n and k are integers with $1 \leq k \leq n$, then $\binom{n}{k} \leq n^k/2^{k-1}$.
- 6*. Prove the hockeystick identity:

$$\sum_{k=0}^r \binom{n+k}{k} = \binom{n+r+1}{r}$$

whenever n and r are positive integers.

More counting

7. How many ways are there to distribute
 - (a) 10 distinguishable balls into four distinguishable bins?
 - (b) 10 indistinguishable balls into four distinguishable bins?
8. How many different combinations of pennies, nickels, dimes, quarters, and half dollars can a piggy bank contain if it has 14 coins in it?
- 9*. How many solutions are there to the inequality $x_1 + x_2 + x_3 + x_4 \leq 15$, where x_1, x_2, x_3, x_4 are nonnegative integers?
- 10 (extra challenge). How many ways are there to distribute 10 distinguishable objects into four distinguishable boxes so that the boxes have one, two, three, and four objects in them, respectively?